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Invariant and attracting set of fuzzy cellular neural networks with variable delays

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ABSTRACT

The aim of this paper is to study the invariant and attracting set of fuzzy cellular neural networks with variable delays. Based on a delayed differential inequality and the properties fuzzy logic operation and M-matrix, the invariant and attracting set is obtained. Moreover, two examples are given to illustrate the effectiveness of our theoretical result.

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1. Introduction

In 1988, Chua and Yang proposed a circuit architecture, called cellular neural network (CNN) [1], which possesses the ability to do parallel signal processing in real time. Compared with general neural networks, CNNs are much more amenable to Very Large Scale Integration implementation according to its neighbor interactive property. Some rather promising applications of CNNs in image processing, communication systems and optimization problem have been reported in [2–4].

In 1996, Yang et al. introduced fuzzy cellular neural network (FCNN), which integrates fuzzy logic into the structure of traditional CNN and maintains local connectedness among cells [5,6]. Unlike previous CNN structures, FCNN has fuzzy logic between its template and input and/or output besides the "sum of product" operation, which allows us to combine the low information processing capability of CNN with the high level information processing capability, such as image understanding of fuzzy systems. FCNN is a useful paradigm for image processing problems and Euclidean distance transformation [7]. Also, FCNN has inherent connection to mathematical morphology, which is a cornerstone in image processing and pattern recognition. To guarantee that the performance of FCNN is what we want, it is important to study its equilibrium points and the stability of those equilibrium points. So various interesting results on the stability of FCNN have been reported (see, for instance, [8–11]). However, if global stability of the equations does not exist, how far can initial conditions be allowed to vary without disrupting the stability properties established in the immediate vicinity of equilibrium states?

Motivated by the idea, one also should discuss the invariant and attracting set of the equations with delays. And the invariant and attracting set of FCNN has not been considered prior to this work, although there has recently been increasing interest in the study of the invariant and attracting set of dynamical systems, and many authors have obtained some results about the problem for differential (or difference) equations with delays (see [12–16]).

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Hence, this paper intends to develop techniques and methods for determining the invariant and attracting set of the following FCNN with variable delays:

$$\begin{cases} \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = -a_i x_i(t) + \displaystyle\sum_{j=1}^{n} b_{ij}\mu_j + I_i \\ \quad + \displaystyle\bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t)) + \bigwedge_{j=1}^{n} \gamma_{ij} f_j(x_j(t-\tau(t)))\mathrm{d}s + \bigwedge_{j=1}^{n} T_{ij}\mu_j \\ \quad + \displaystyle\bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t)) + \bigvee_{j=1}^{n} \theta_{ij} f_j(x_j(t-\tau(t)))\mathrm{d}s + \bigvee_{j=1}^{n} H_{ij}\mu_j, \\ x_i(t_0+s) = \phi_i(s), \quad -\tau \le s \le 0, \end{cases} \tag{1}$$

where $i = 1, \ldots, n, a_i > 0, \alpha_{ij}$ and γ_{ij} are elements of fuzzy feedback MIN template; β_{ij} and θ_{ij} are elements of fuzzy feedback MAX template; T_{ij} and H_{ij} are elements of the fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively; b_{ij} are elements of the feed-forward template; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $x_i,\ \mu_i,\ I_i$ denote the state, input, bias of the ith neurons, respectively; and $0 \le \tau(t) \le \tau$, where τ is a constant.

2. Preliminaries

In what follows, we will introduce some notations and basic definitions.

Let R^n be the space of n-dimensional real column vectors and $R^{m\times n}$ denote the set of $m \times n$ real matrices. Usually E denotes an $n \times n$ unit matrix. For $A, B \in R^{m\times n}$ or $A, B \in R^n, A \ge B (A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\ge$ ($>$)". In particular, A is called a nonnegative matrix if $A \ge 0$, and z is called a positive vector if $z > 0$.

$C[X, Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. In particular, let $C \triangleq C[(-\infty, 0], R^n]$.

For $x \in R^n$, $A \in R^{n\times n}$, we define $[x]^+ = (|x_1|, \ldots, |x_n|)^{\mathrm{T}}$, $[A]^+ = (|a_{ij}|)_{n\times n}$, for $\phi \in C$, $[\phi(t)]_\tau = ([\phi_1(t)]_\tau, \ldots, [\phi_n(t)]_\tau)^{\mathrm{T}}$, $[\phi(t)]_\tau^+ = [[\phi(t)]^+]_\tau$, where $[\phi_i(t)]_\tau = \sup_{-\tau\le s\le 0}\{\phi_i(t+s)\}$. For convenience, we denote

$$\|x\| = \max_{1\le i\le n}\{|x_i|\}, \qquad \|A\| = \max_{1\le i\le n}\sum_{j=1}^{n}|a_{ij}|, \qquad \|\phi\| = \max_{1\le i\le n}\{[\phi_i(t)]_\tau^+\}.$$

Definition 1. FCNN (1) is said to be uniformly dissipative, if there is a constant $N > 0$, such that for any solution $x(t) = (x_1(t), \ldots, x_n(t))^{\mathrm{T}}$ of FCNN (1), one has $\lim_{t\to\infty}\sup|x_i(t)| \le N, i = 1, \ldots, n$.

Definition 2. The set $S \subset C$ is called a positive invariant set of (1) if, for any initial value $\phi \in S$, the solution $x(t, t_0, \phi) \in S$, for $t \ge t_0$.

Definition 3. A set $S_1 \subset C$ is called a global attracting set of (1) if, for any initial value $\phi \in D$, the solution $x(t, t_0, \phi)$ converges to S_1 as $t \to +\infty$; that is,

$$\mathrm{dist}(x(t, t_0, \phi), S_1) \to 0, \quad t \to +\infty, \tag{2}$$

where $\mathrm{dist}(x, S_1) = \inf_{\psi\in S_1} d(x, \psi)$ and $d(x, \psi) = \sup_{-\tau\le s\le 0}|x(s) - \psi(s)|$.

Definition 4 ([17]). Let the matrix $D = (d_{ij})_{n\times n}$ has nonpositive off-diagonal elements (*i.e.*, $d_{ij} \le 0, i \ne j$); then each of the following conditions is equivalent to the statement "D is a nonsingular M-matrix".

(i) All the leading principal minors of D are positive.

(ii) D is inverse-positive; that is, D^{-1} exists and $D^{-1} \ge 0$.

(iii) The diagonal elements of D are all positive and there exists a positive vector d such that $Dd > 0$ or $D^{\mathrm{T}}d > 0$.

For a nonsingular M-matrix D, we denote

$$\Omega_M(D) \triangleq \{z \in R^n | Dz > 0, z > 0\}.$$

From (iii) of the definition of an M-matrix, we have the following lemma.

Lemma 1. *$\Omega_M(D)$ is nonempty and for any $z_i \in \Omega_M(D)$ $(i = 1, \ldots, n)$, we have*

$$k_1 z_1 \in \Omega_M(D) \quad \textit{and} \quad \sum_{i=1}^{n} k_i z_i \in \Omega_M(D), \quad \forall\, k_i > 0.$$

Lemma 2 ([6]). *For any $a_{ij} \in R$, $x_j, y_j \in R$, $i, j = 1, \ldots, n$, we have the following estimations:*

$$\left| \bigwedge_{j=1}^{n} a_{ij}x_j - \bigwedge_{j=1}^{n} a_{ij}y_j \right| \le \sum_{j=1}^{n} |a_{ij}||x_j - y_j|, \tag{3}$$

and

$$\left| \bigvee_{j=1}^{n} a_{ij}x_j - \bigvee_{j=1}^{n} a_{ij}y_j \right| \le \sum_{j=1}^{n} |a_{ij}||x_j - y_j|. \tag{4}$$

3. Main results

Theorem 1. *Let $0 \le u(t) = (u_1(t), \ldots, u_n(t))^{\mathrm{T}} \in C[[t_0, \infty), R^n]$ satisfy the following delayed differential inequality*

$$\begin{cases} D^+u(t) \le Pu(t) + Q[u(t)]_\tau + J, & t \ge t_0, \\ u(t) = \phi(t), & t_0 - \tau \le t \le t_0, \end{cases} \tag{5}$$

where $P = (p_{ij})_{n\times n}$ with $p_{ij} \ge 0$ for $i \ne j$, $Q = (q_{ij})_{n\times n} \ge 0$, $J = (J_1, \ldots, J_n)^{\mathrm{T}} \ge 0$, $\phi(t) \in C[[t_0 - \tau, t_0], R^n]$.

If $D = -(P + Q)$ is a nonsingular M-matrix, then there exists a positive vector $z = (z_1, \ldots, z_n)^{\mathrm{T}} \in \Omega_M(D)$ such that

$$u(t) \le z\mathrm{e}^{-\lambda(t-t_0)} - (P + Q)^{-1}J, \quad t \ge t_0, \tag{6}$$

where the positive constant λ is determined by the following inequality:

$$[\lambda E + P + Q\mathrm{e}^{\lambda\tau}]z < 0, \quad \text{for the given } z \in \Omega_M(D). \tag{7}$$

Proof. Since D is a nonsingular M-matrix, there exists a positive vector $z \in \Omega_M(D)$ such that $Dz > 0$ or $(P + Q)z < 0$. By using continuity, we know that there exists at least one positive constant λ such that (7) holds, i.e.,

$$\lambda z_i + \sum_{j=1}^{n} \left(p_{ij} + q_{ij}\mathrm{e}^{\lambda\tau}\right) z_j < 0. \tag{8}$$

Since $\phi(t) \in C[[t_0 - \tau, t_0], R^n]$ is bounded, by Lemma 1, we can choose a sufficiently large $z \in \Omega_M(D)$ such that

$$u(t) \le z\mathrm{e}^{-\lambda(t-t_0)} - (P + Q)^{-1}J, \quad t \in (t_0 - \tau, t_0]. \tag{9}$$

Next we shall prove that (6) holds for any $t \ge t_0$.

Let $H = -(P + Q)^{-1}J$, $H = (H_1, \ldots, H_n)^{\mathrm{T}}$; then we have

$$\sum_{j=1}^{n} p_{ij}H_j + \sum_{j=1}^{n} q_{ij}H_j + J_i = 0, \quad i = 1, \ldots, n. \tag{10}$$

In order to prove (6), we first prove for any constant $l > 1$,

$$u_i(t) < l(z_i\mathrm{e}^{-\lambda(t-t_0)} + H_i) \triangleq v_i(t), \quad t \ge t_0,\ i = 1, \ldots, n. \tag{11}$$

If (11) is not true, from the fact that $u_i(t)$ is continuous, then there must be a $t^* > t_0$ and some integer m such that

$$u_m(t^*) = v_m(t^*), \quad D^+u_m(t^*) \ge v'_m(t^*), \tag{12}$$

$$u_i(t) \le v_i(t), \quad t \in (t_0 - \tau, t^*],\ i = 1, \ldots, n. \tag{13}$$

Hence, by (5) and (8), the equality of (12) and (13), and $p_{ij} \ge 0$ $(i \ne j)$, $q_{ij}(t) \ge 0$, we derive that

$$\begin{aligned} D^+u_m(t^*) &\le \sum_{j=1}^{n} p_{mj}u_j(t^*) + \sum_{j=1}^{n} q_{mj}[u_j(t^*)]_\tau + J_m \\ &\le \sum_{j=1}^{n} \left(p_{mj} + q_{mj}\mathrm{e}^{\lambda\tau}\right) lz_j\mathrm{e}^{-\lambda(t^*-t_0)} + \sum_{j=1}^{n} \left(p_{mj} + q_{mj}\right) lH_j + J_m \\ &= \sum_{j=1}^{n} \left(p_{mj} + q_{mj}\mathrm{e}^{\lambda\tau}\right) lz_j\mathrm{e}^{-\lambda(t^*-t_0)} - (l-1)J_m \end{aligned}$$

$$\begin{aligned}
&\le \sum_{j=1}^{n}\left(p_{mj}+q_{mj}\mathrm{e}^{\lambda\tau}\right) l z_j \mathrm{e}^{-\lambda(t^*-t_0)}\\
&< -\lambda z_m l \mathrm{e}^{-\lambda(t^*-t_0)}\\
&= v_m'(t^*),
\end{aligned}$$

which contradicts the inequality of (12), so (11) holds for all $t \ge t_0$. Letting $l \to 1$, then we have

$$u(t) \le z\mathrm{e}^{-\lambda(t-t_0)} - (P+Q)^{-1}J, \quad t \ge t_0.$$

This completes the proof. □

Next, we will state our main results. For convenience, we denote

$$A_0 = \mathrm{diag}\{a_1, \ldots, a_n\}, \qquad A = (|\alpha_{ij}| + |\beta_{ij}|)_{n\times n},$$

$$B = (|\gamma_{ij}| + |\theta_{ij}|)_{n\times n}, \qquad J = \mathrm{col}\left\{I_i + \sum_{j=1}^{n}(|b_{ij}|\cdot|\mu_j| + |T_{ij}|\cdot|\mu_j| + |H_{ij}|\cdot|\mu_j|)\right\}_{i=1}^{n}.$$

Throughout the paper, we assume that

(H1) For all $s \in R, j = 1, \ldots, n$, there exist nonnegative constants L_j such that

$$0 \le \mathrm{sgn}(s) f_j(s) \le L_j|s|,$$

where $\mathrm{sgn}(\cdot)$ is sign function;

(H2) $D = -(P+Q)$ is a nonsingular M-matrix, where $P = -A_0 + AL$, $Q = BL$, $L = \mathrm{diag}\{L_1, \ldots, L_n\}$.

Theorem 2. *Suppose that Conditions* (H1) *and* (H2) *are satisfied, then FCNN* (1) *is uniformly dissipative.*

Proof. Calculating the upper right derivative $D^+|x_i(t)|$ along the solution of (1), from (1), (H_1) and Lemma 2, we can get

$$\begin{aligned}
D^+|x_i(t)| &= \mathrm{sgn}(x_i(t))\frac{\mathrm{d}x_i}{\mathrm{d}t}\\
&\le -a_i|x_i(t)| + \left|\bigwedge_{j=1}^{n}\alpha_{ij}f_j(x_j(t))\right| + \left|\bigwedge_{j=1}^{n}\gamma_{ij}f_j(x_j(t-\tau(t)))\right|\\
&\quad + \left|\bigvee_{j=1}^{n}\beta_{ij}f_j(x_j(t))\right| + \left|\bigvee_{j=1}^{n}\theta_{ij}f_j(x_j(t-\tau(t)))\right| + \left|\sum_{j=1}^{n}b_{ij}\mu_j\right| + \left|\bigwedge_{j=1}^{n}T_{ij}\mu_j\right| + \left|\bigvee_{j=1}^{n}H_{ij}\mu_j\right| + |I_i|\\
&\le -a_i|x_i(t)| + \sum_{j=1}^{n}\left(|\alpha_{ij}| + |\beta_{ij}|\right)|f_j(x_j(t))|\\
&\quad + \sum_{j=1}^{n}\left(|\gamma_{ij}| + |\theta_{ij}|\right)|f_j(x_j(t-\tau(t)))| + |I_i| + \sum_{j=1}^{n}(|b_{ij}|\cdot|\mu_j| + |T_{ij}|\cdot|\mu_j| + |H_{ij}|\cdot|\mu_j|)\\
&\le -a_i|x_i(t)| + \sum_{j=1}^{n}\left(|\alpha_{ij}| + |\beta_{ij}|\right)L_j|x_j(t)|\\
&\quad + \sum_{j=1}^{n}\left(|\gamma_{ij}| + |\theta_{ij}|\right)L_j|x_j(t-\tau(t))| + |I_i| + \sum_{j=1}^{n}(|b_{ij}|\cdot|\mu_j| + |T_{ij}|\cdot|\mu_j| + |H_{ij}|\cdot|\mu_j|).
\end{aligned} \tag{14}$$

Then from (14) and noting the conditions (H1) and (H2), we have

$$\begin{aligned}
D^+[x(t)]^+ &\le -A_0[x(t)]^+ + AL[x(t)]^+ + BL[y(t)]_\tau^+ + J\\
&= P[x(t)]^+ + Q[x(t)]_\tau^+ + J, \quad t \ge t_0.
\end{aligned} \tag{15}$$

Employing Theorem 1, FCNN (1) is uniformly dissipative. □

By Theorem 2, we can easily obtain the following results.

Theorem 3. *Assume that Conditions* (H1) *and* (H2) *hold. Then* $S = \{\phi \in C | [\phi]_\tau^+ \le H = -(P+Q)^{-1}J\}$ *is the positive invariant set of FCNN* (1).

Theorem 4. *Assume that Conditions* (H1) *and* (H2) *hold. Then* $S = \{\phi \in C | [\phi]_\tau^+ \le H = -(P+Q)^{-1}J\}$ *is the global attracting set of FCNN* (1).

Proof. Let $x(t) = (x_1(t), \ldots, x_n(t))^{\mathrm{T}}$ be an arbitrary solution of (1) and select an arbitrary $\psi \in S$; we will show that for any initial value $\phi \in C$,

$$dist(x(t), S) = \inf_{\psi \in S} d(x(t), \psi) \to 0, \quad \text{as } t \to \infty. \tag{16}$$

Denote $y(t) = x(t) - \psi, y(t) = (y_1(t), \ldots, y_n(t))^{\mathrm{T}}$; then calculating the upper right derivative $D^+|y_i(t)|$ along the solution of (1),

$$\begin{aligned} D^+|y_i(t)| &= D^+|x_i(t) - \psi| \\ &\le -a_i|x_i(t)| + \sum_{j=1}^{n} \left(|\alpha_{ij}| + |\beta_{ij}|\right) L_j|x_j(t)| + \sum_{j=1}^{n} \left(|\gamma_{ij}| + |\theta_{ij}|\right) L_j|x_j(t-\tau(t))| \\ &\quad + |I_i| + \sum_{j=1}^{n} (|b_{ij}| \cdot |\mu_j| + |T_{ij}| \cdot |\mu_j| + |H_{ij}| \cdot |\mu_j|). \end{aligned} \tag{17}$$

Then from (17) and noting the conditions (H1) and (H2), we obtain that

$$\begin{aligned} D^+[y(t)]^+ &= D^+[x(t) - \psi]^+ \\ &\le P[x(t)]^+ + Q[x(t)]_\tau^+ + J \\ &\le P[x(t) - \psi]^+ + Q[x(t) - \psi]_\tau^+ + P\psi + Q\psi + J \\ &\le P[y(t)]^+ + Q[y(t)]_\tau^+. \end{aligned} \tag{18}$$

Then from Theorem 1 with $J = 0$, we derive that

$$[y(t)]^+ \le z\mathrm{e}^{-\lambda(t-t_0)}, \quad t \ge t_0, \tag{19}$$

where $\lambda > 0$ is determined by

$$[\lambda E + P + Q\mathrm{e}^{\lambda\tau}]z < 0, \quad \text{for the given } z \in \Omega_M(D). \tag{20}$$

So (16) holds from (19). The proof is completed. □

4. Two illustrative examples

Example 1. Consider FCNN (1) with the following parameters and activation functions ($n = 2, i, j = 1, 2$).

$$\begin{aligned} &a_1 = 10, \quad a_2 = 8, \quad \alpha_{11} = 1, \quad \alpha_{12} = 2, \quad \alpha_{21} = 3, \quad \alpha_{22} = 1, \quad \beta_{11} = \beta_{12} = 1, \\ &\beta_{21} = \beta_{22} = 2, \quad \gamma_{ij} = 1, \quad \theta_{11} = 1, \quad \theta_{12} = 2, \quad \theta_{21} = 2, \quad \theta_{22} = 3, \quad b_{ij} = 1, \quad T_{ij} = 1, \\ &H_{ij} = 1, \quad \mu_j = 1, \quad I_1 = 1, \quad I_2 = 2, \quad f_j(x_j) = \tanh(x_j), \quad \tau(t) = \mathrm{e}^{-t} \le 1, \quad t \ge 0. \end{aligned} \tag{21}$$

From the given parameters, we have

$$A_0 = \begin{pmatrix} 10 & 0 \\ 0 & 8 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 3 \\ 5 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad J = \begin{pmatrix} 7 \\ 8 \end{pmatrix}.$$

So we obtain

$$P = \begin{pmatrix} -8 & 3 \\ 5 & -5 \end{pmatrix}, \quad Q = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 7 & -4 \\ -6 & 4 \end{pmatrix},$$

where $P = -A_0 + AL, D = -(P + Q)$. We can easily get that D is a nonsingular M-matrix. Therefore, by Theorems 3 and 4, we can conclude that

$$S = \left\{ \phi \in C | [\phi]^+ \le W = -(P + Q)^{-1}J = \left(15, \frac{49}{2}\right)^{\mathrm{T}} \right\} \tag{22}$$

is the positively invariant and globally attracting set of FCNN (1) with the parameters given by (21).

Example 2. Consider the parameters and activation functions of FCNN (1) as follows ($n = 2, i, j = 1, 2$):

$$a_1 = 27, \quad a_2 = 13, \quad \alpha_{11} = \frac{1}{4}, \quad \alpha_{12} = 2, \quad \alpha_{21} = 1, \quad \alpha_{22} = \frac{1}{4}, \quad \beta_{11} = \frac{1}{4}, \quad \beta_{12} = 4,$$

$$\beta_{21} = \frac{1}{2}, \quad \beta_{22} = \frac{3}{4}, \quad \gamma_{11} = 2, \quad \gamma_{12} = 1, \quad \gamma_{21} = 2, \quad \gamma_{22} = 3, \quad \theta_{11} = 2, \quad \theta_{12} = 1,$$
$$\theta_{21} = 4, \quad \theta_{22} = 1, \quad b_{ij} = 1, \quad T_{ij} = 1, \quad H_{ij} = 1, \quad \mu_j = 1, \quad I_1 = 7, \quad I_2 = -2,$$
$$f_j(x_j) = \frac{2}{j}|x_j|, \text{ which satisfies (H1) with } L_j = \frac{2}{j}, \quad \tau(t) = |\cos t| \le 1, \quad t \ge 0. \tag{23}$$

From the given parameters, we have

$$A_0 = \begin{pmatrix} 27 & 0 \\ 0 & 13 \end{pmatrix}, \quad A = \begin{pmatrix} \frac{1}{2} & 6 \\ \frac{3}{2} & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 \\ 6 & 4 \end{pmatrix}, \quad L = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \quad J = \begin{pmatrix} 13 \\ 4 \end{pmatrix}.$$

So

$$P = -A_0 + AL = \begin{pmatrix} -26 & 6 \\ 3 & -11 \end{pmatrix}, \quad Q = \begin{pmatrix} 8 & 2 \\ 12 & 4 \end{pmatrix}, \quad D = -(P+Q) = \begin{pmatrix} 18 & -8 \\ -15 & 7 \end{pmatrix}.$$

We can easily obtain that D is a nonsingular M-matrix. By Theorems 2 and 3, we can conclude that

$$S = \left\{ \phi \in C | [\phi]^+ \le W = -(P+Q)^{-1}J = \left(\frac{123}{6}, \frac{89}{2} \right)^{\mathrm{T}} \right\} \tag{24}$$

is the positively invariant and globally attracting set of FCNN (1) with the parameters given by (23).

5. Conclusion

In this paper, by establishing an delayed differential inequality and using M-matrix theory, several simple sufficient conditions on the invariant and attracting set of FCNNs with variable delays have been obtained. The results obtained generalize and improve some of the existing results on the invariant and attracting set of delayed neural networks in [12–16]. Moreover, examples are given to show the effectiveness of our theoretical results.

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